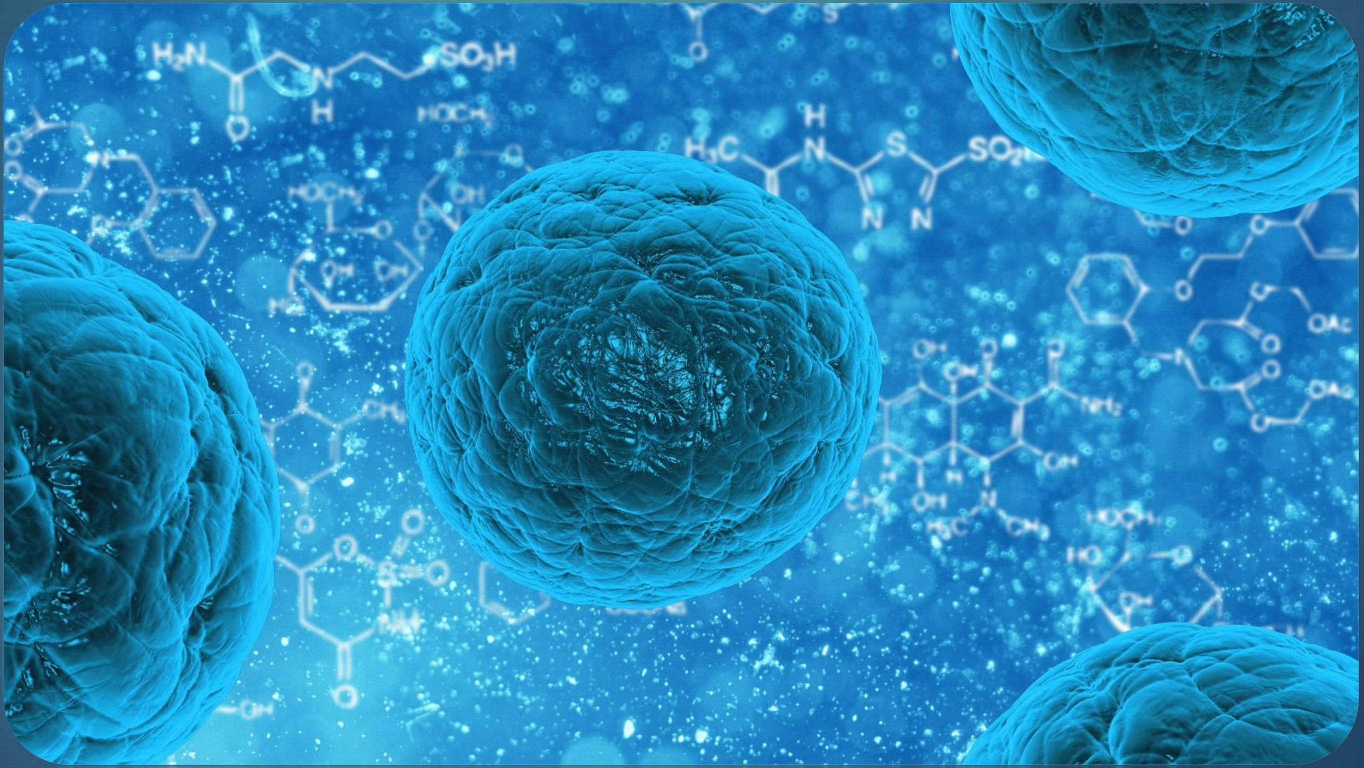
In the name of God

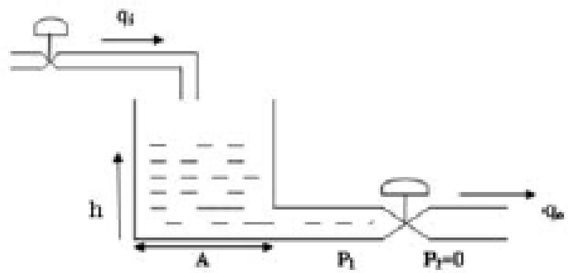
Modelling biological systems

S.Ali.Zendehbad



Example of Hydraulic system

- Draw the electrical model for this system

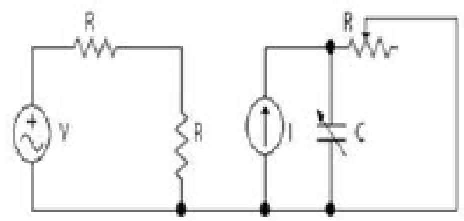


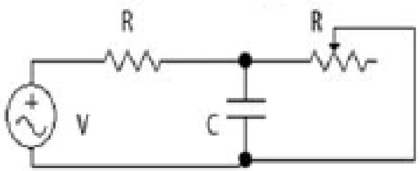
- solution

$$\begin{cases} q_c = q_i - q_o \\ V = A.h \end{cases} \Longrightarrow q_c = q_i - q_o = \frac{dV}{dt} = A\frac{dh}{dt}$$

Example of Hydraulic system

- solution





If q_i is produced with a supplier we can put the current supply directly

If production of the q_i is influenced by pressure we should use Thevenin's Theorem

Thermal System

- Primary variables
  - Longitudinal variables : heat flow (or thermal power)
  - Transverse variables : temperature
- Secondary variables
  - Thermal resistance
  - Thermal Capacitors

Thermal System

$$electrical: R = \frac{volt}{ampere} = \Omega \quad \longrightarrow \quad thermal: R_{th} = \frac{temperature}{power} = \frac{\Delta T}{P} = \frac{\circ C}{w}$$

$$energy: Q = MC(T_2 - T_1) \quad \longrightarrow \quad power: P = \frac{dQ}{dt} = MC\frac{d\Delta T}{dt}$$

- Thermal systems don’t have inductor so these systems only include RC circuit
- If heat is pass through different layer the order of the system is defined by number of the layers. Like heat sinks that is used for transistors

Modeling Analogies

- Two classes of variables

System	Flow variable	Effort variable
Electric	current	voltage
Translational	velocity	force
Rotational	angular velocity	torque
Hydraulic	volume flow	pressure difference
Thermal	heat flow	temperature

Traffic System

- Primary variables
  - Longitudinal variables : traffic flow
  - Transverse variables : traffic pressure
- Secondary variables
  - Traffic resistance : narrow way
  - Traffic capacitor : parking
  - Traffic inductor : automobile inertia
  - ON-Off switch : traffic light

These variables were introduced just as a suggestion

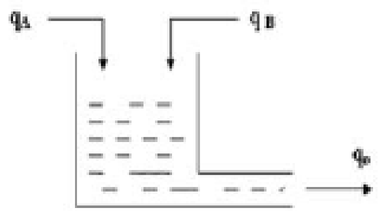
Psychology System

- Used for modeling of the human behavior
- Suggested variables :
  - Mental pressure
  - Mental capacitance
  - Stress
  - motivation

Chemical System

- Used for modeling the chemical reaction and fluidity behavior of system
- In these models usually use volume flow for study fluids and mass flow for input material to system
- Example:

  Write the mathematical equation for shown system



Chemical System

- **Definition**

$q:\ Volume\ Flow$

$c:\ Volume\ \text{Density}$

$\text{q}_\text{o}:Output\ Volume\ Flow$

$V:Total\ Volume$

$r_A^+:\text{Pr}\,oduction\ Rate\ of\ A$

$r_A^-:\text{Diminution}\ Rate\ of\ A$

$r_A = r_A^+ - r_A^-$

Chemical System

- **Solution**

$$\alpha A + \beta B \leftrightarrow nD$$

- **If** $\alpha = \beta = 1$ **then** $A + B \leftrightarrow nD$ $r_A = r_B$

$$r_D = nr_A = nr_B = nr = r_D^{+} - r_D^{-}$$

$$q_A \cdot \rho_A - q_o c_A - rV = \frac{d(C_A V)}{dt}$$

$$q_B \rho_B - q_o c_B - rV = \frac{d(C_B V)}{dt}$$

Linear Graph

- A graphical method for modeling and evaluating complex and mixed systems
- The same representation for all systems in any field (electrical, mechanical,…)

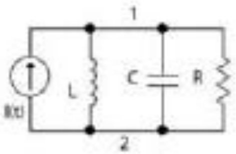
- The first and important stage of this method :
  - determine the subsystem
  - Determine the interconnection in each subsystem

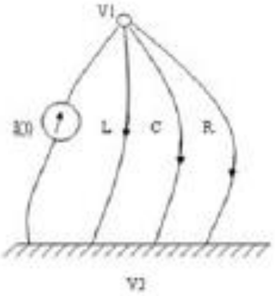
- Electrical Machine: electric system, mechanic system,…

- For Example In the mechanical system connection of elements are considered as interconnection point. In these systems because all of the particle of the mass have the same velocity therefore mass element aren't located between two interconnection points and connected to ground

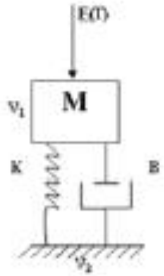
Examples

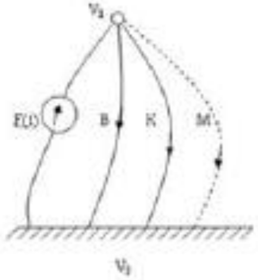
Example1:



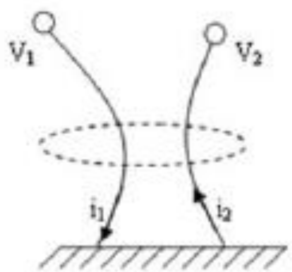


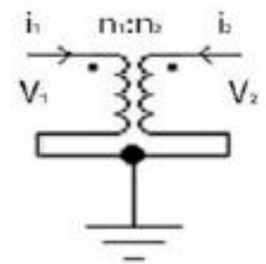
Example2:





Transformer



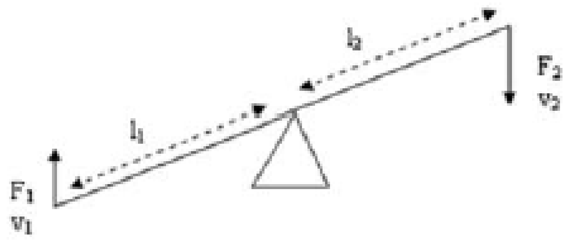


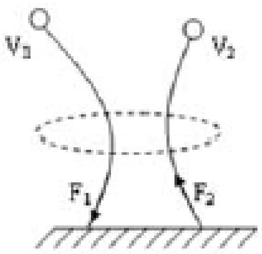
$$\frac{V_1}{V_2} = \frac{n_1}{n_2} = \frac{1}{n}$$

$$V_1 i_1 = V_2 i_2 \longrightarrow \frac{V_1}{V_2} = \frac{i_2}{i_1} = \frac{1}{n}$$

Lever

- Levers act as transformers of force and velocity

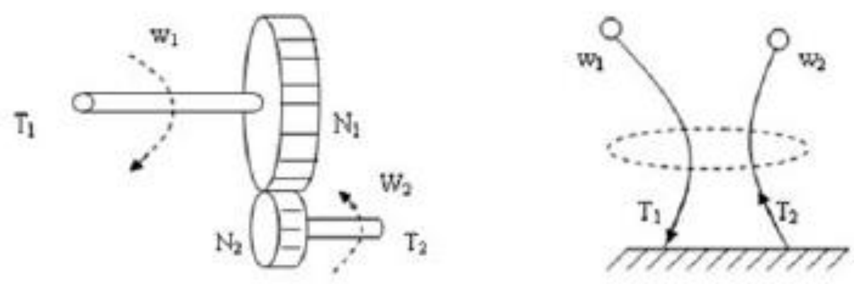




$$\left.\begin{array}{l} T_1 = T_2 \\ F_1 l_1 = F_2 . l_2 \\ \dfrac{V_2}{V_1} = \dfrac{l_2}{l_1} \end{array}\right\} \Rightarrow \frac{V_2}{V_1} = \frac{F_1}{F_2} = \frac{l_2}{l_1} = n$$

Gear Box

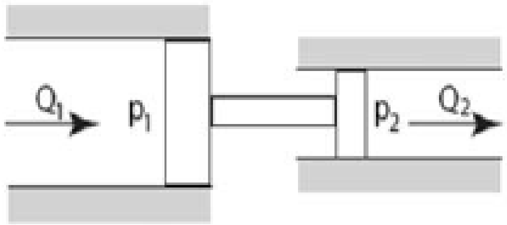
- Gear boxes transforms torques and angular velocity



$$P_1 = P_2$$

$$T_1.w_1 = T_2.w_2 \qquad \frac{w_2}{w_1} = \frac{T_1}{T_2} = \frac{N_2}{N_1} = n$$

Hydraulic Transformer



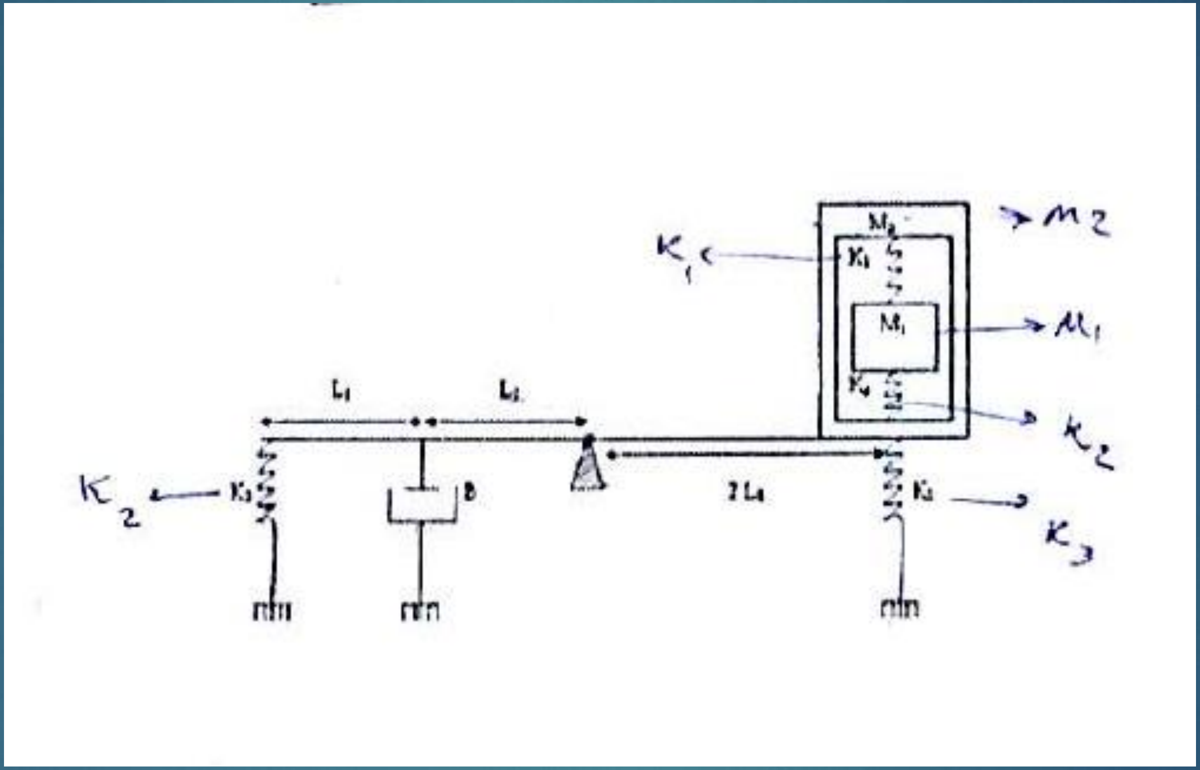
$$F_1 = F_2$$

$$P_1 A_1 = P_2 A_2$$

$$\frac{P_1}{P_2} = \frac{A_2}{A_1} = n$$

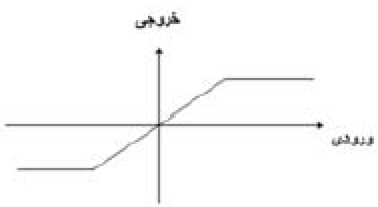
$$P_1 q_1 = P_2 q_2$$

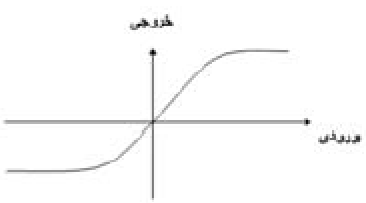
$$\frac{P_1}{P_2} = \frac{q_2}{q_1} = n$$



Nonlinear considerations in modeling

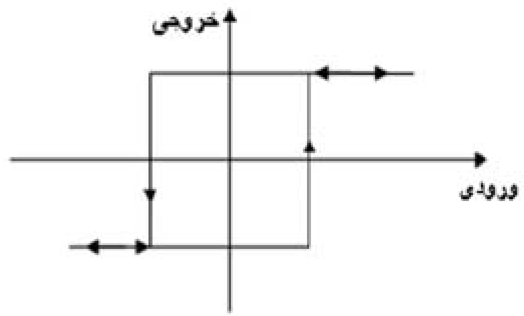
- Saturation
  - Occur when output doesn’t trace input like op-amp
  - One of the modeling mathematical function of saturation is sigmoid that is used in artificial neural network





Nonlinear considerations in modeling

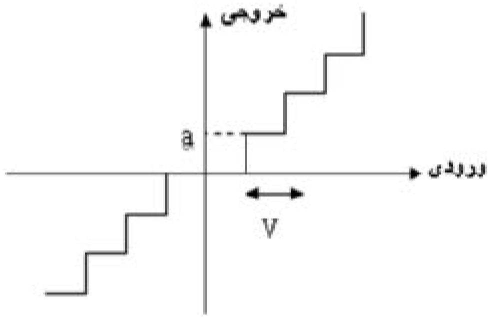
- Hysteresis
  - Occur when forward and backward path of input-output are not the same



Hysteresis with saturation in system

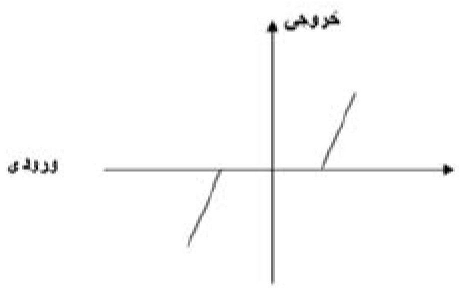
Nonlinear considerations in modeling

- Quantization
  - Occur in digitizing analog signal and proportional to the number of bit of A/D



Nonlinear considerations in modeling

- Dead zone
  - Described base on production of output when input reaches the special threshold
  - like the current-voltage of diode , friction , …



Lumped systems

- The functions just depend on time
- Location, distance and dimension of elements haven't influence on the functions. In other words all of the component in any location see the flow wave at the same time.
- For example
  - If the frequency of a circuit is 1kHz then $\lambda = \frac{c}{f} = \frac{3\times10^8}{1000} = 300\text{km}$

    so if the components are in the centimeter range they will receive the wave at the same time and the location aren't important.

Distributed systems

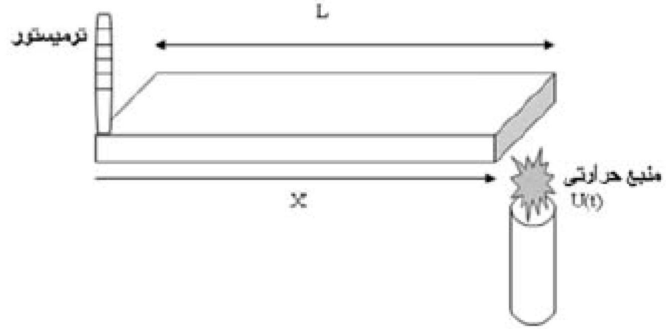
- In these systems the functions depend on time and location or generally depend on several parameter.
- For example
  - In a communicational system if the frequency of antenna is 100MHz then

$$\lambda = \frac{3 \times 10^8}{100 \times 10^6} = 3\text{m}$$

    so we can see that the length of antenna is important and the flow signal aren't only function of time and depend on length too.
- Thermal and mechanical wave (vibration) transfer systems are extended system
- Differential equation with partial derivation are cross section of these systems.

Example

- **Determine the equation of heat transfer from source to thermistor.**



Example

- solution

$$\frac{\partial h(t,x)}{\partial t} = k\frac{\partial^2 h(t,x)}{\partial x^2} \quad k : heat\ conduction\ \text{Coefficien t}$$

$$\begin{cases} \left.\frac{\partial h(t,x)}{\partial x}\right|_{x=L} = KU(t) \quad K : Heat\ transfer\ \text{Coefficien t} \\ \left.\frac{\partial h(t,x)}{\partial x}\right|_{x=0} = 0 \end{cases}$$

✓Numerical method

✓Laplace method

Example

- ✓ **Numerical method**

$$\frac{\partial^2 h(t,x)}{\partial x^2} = \frac{h(t,x+\Delta x) - 2h(t,x) + h(t,x-\Delta x)}{\Delta x^2}$$

- ✓ **Laplace method**

$$\mathrm{SH(s,x) = kH''(s,x)}$$

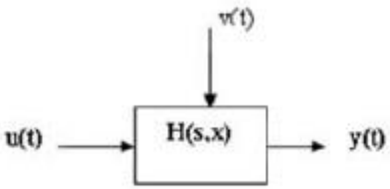
$$\text{Boundary Condition} : H'(s,x)\big|_{x=L} = KU(s) \ , H'(s,x)\big|_{x=0} = 0$$

$$H(s,x) = A(s)e^{-x\sqrt{\frac{s}{k}}} + B(s)e^{x\sqrt{\frac{s}{k}}}$$

$$A(s) = B(s) = \frac{kU(s)}{\sqrt{\frac{s}{k}}(e^{L\sqrt{\frac{s}{k}}} - e^{-L\sqrt{\frac{s}{k}}})}$$

Example

- This system is considered for Thermistor



$$y(t) = h(t,0) * u(t) + v(t) \qquad v(t): noise\ function$$

$$Y(s) = H(s,x)\big|_{x=0} U(s) + V(s)$$

$$Transfer\ Function\ of\ Thermistor: G(s) = \frac{H(s,x)\big|_{x=0}}{U(s)} = \frac{2k}{\sqrt{\frac{s}{k}}\left(e^{L\sqrt{\frac{s}{k}}} - e^{-L\sqrt{\frac{s}{k}}}\right)}$$

contact us

E-mail : Ali.zendebad@gmail.com

Homepage:

Sazendehbad.ir

Telegram:

@Cyberstudents


obrigado
Dank U
Merci
mahalo
Köszi
спасибо
Grazie
Thank you
mauruuru
Takk
Gracias
Dziękuję
Děkuju
danke
Kiitos